TABLES OF STATISTICAL ERROR*

BY

PROFESSOR SIR RONALD ROSS, K.C.B., F.R.S.,

AND

WALTER STOTT,

HONORARY STATISTICIAN, LIVERPOOL SCHOOL OF TROPICAL MEDICINE

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I. EXPLANATION

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These Tables have been specially constructed for practical use in sanitary, pathological and clinical work.

Suppose that we are studying things of any nature, such for example as men, cases of sickness, insects, leucocytes, trypanosomes, seeds, stones, etc., and wish to ascertain what proportion of all of them belong to a particular class, that is, have a given characteristic. Then we can answer this question with absolute certainty only in one way, namely, by examining the whole number of such things that exist in the region under consideration. But this is generally impossible; and we must, therefore, content ourself with examining only as many of the things as we can—ascertaining what proportion of these possess the given characteristic, and thence *inferring* what probable proportion of the same things in the region under consideration belong to the same class.

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For example, if we wish to know with certainty how many people of any nationality have blue eyes, then we must examine all the people of that nationality. But this will be impossible. Hence we must examine 10, or 100, or 1,000 or more of the people; ascertain how many of *these* have blue eyes; and then *infer* from this sample what proportion of the whole nation are *likely* to have them.

Everyone knows, of course, that (provided the methods of examination are always equally careful and trustworthy) our estimate is more likely to be near the truth if we examine a large sample than if we examine only a small one—that is, if we examine many of the things than if we examine only a few. For instance, it would be absurd to attempt to estimate the proportion of persons with blue eyes by examining only five or ten persons. We should come nearer by examining one hundred or one thousand, and so on; but we should reach absolute certainty only by examining all the people in the country. The important question now arises: How many of the things must we examine in order to become *reasonably sure* that our result is within a *given percentage* of the exact truth? The labours of mathematicians enable us to answer this question, and the following Tables enable us to answer it in most cases without calculation. We must begin by dealing with the case in which the total number of things is so large that we cannot take for a sample more than a very small proportion of that total number. Hence the heading of this section is Proportionately Small Samples.

First, however, we must understand what exactly we mean by the phrases '*reasonably sure*' and 'a given *percentage* of the truth.' Both these phrases contain ideas of *degree*. Thus, if we are carrying out a strict scientific enquiry, we may wish to be able to say that the betting (or probability, as it is called) is 99,999 to 1 that our result is within one per cent. of the truth. We must then consult Table A 1, under the second column. Or it may suffice to say that the betting is 99,999 to 1 that our result is within five per cent. of the truth; and we then consult the same table under the sixth column. But it may happen that we can afford to be content with a lower degree of probability than this—it may suffice to say that the betting is only 99 to 1 that our result is within 3 per cent. of the truth; and we then look at Table A 4, under the fourth column. Lastly, we may be allowed to content ourself with an 'even chance' or 'toss up'—that is, a probability of 1 to 1; and for this we consult Table A 6.

The reader will now be easily able to understand the tables. The figures in the heading of each give the degree of sureness (so to speak) with which we may rely upon the inferences drawn from the table. Thus the odds are 99,999 to 1 that the inferences to be drawn from the first table are sound; or, in other words, out of 100,000 trials of the table only one is likely to be wrong. Table A 5, however, is likely to be wrong once in ten trials; and Table A 6, once in two.

The percentages at the head of each column give the percentage of 'statistical error'—that is, the amount by which the truth is likely to diverge from our observed result. Thus, if our result is 70 per cent., and the statistical error is 5 per cent., then the odds (as denoted by the figures 99,999 to 1, etc.) are that the truth will lie anywhere between 70 + 5 per cent. and 70 − 5 per cent.—that is, between 75 per cent. and 65 per cent. If, however, the statistical error is only one per cent., then the truth is likely to lie between the narrower limits, 71 per cent. and 69 per cent. If the observed result, *plus* the error, exceeds 100, or if the observed result, minus the error, is less than 0, we conclude that the number of things hitherto examined is not yet large enough to yield a useful result.

The figures down the first (left hand) column of each table refer to the percentage of the observed result up to 50 per cent.; and the figures on the same line in the body of the table give the total number of things which must be examined in order to ensure that that result will lie within the percentage of error given at the head of the corresponding column, with the degree of probability given at the head of each table. Thus, if the observed result is 43 per cent., we must examine 47,780 things to ensure that the odds are 99,999 to 1 that the truth lies between 44 per cent. and 42 per cent. But we need examine only 1,328 things to ensure that the betting is 99,999 to 1 that the truth lies between 49 per cent. and 37 per cent. We must examine 26,541 things to ensure, with a probability of 999 to 1, that the truth lies between 44 per cent. and 42 per cent. (Table A 3); but we need examine only 415 things to ensure, with a probability of 9 to 1, that the truth lies between 47 per cent. and 39 per cent. (Table A 5); and only 185 things that, with a probability of 9 to 1, it lies between 49 per cent. and 37 per cent.

If the observed percentage is over 50, we subtract it from 100 and obtain the required figures for the remainder. For example, the figures for observed percentages of 70 per cent., 79 per cent., and 85 per cent. are precisely the same as those for 30 per cent., 21 per cent. and 15 per cent.

On inspecting the tables we see at once that the figures diminish rapidly in successive tables, and also in successive columns (from left to right). From these observations we gather, as we might have expected, that the number of things required to be examined diminishes (*a*) with reduced probability of correctness, and (*b*) with increased statistical error.

The figures increase as we descend the columns. That is, the number of things required to be examined increases as the observed percentage rises from 1 per cent. to 50 per cent.; but after that (by the rule just given) it diminishes as the observed percentage continues to rise from 50 per cent. to 100 per cent.

The tables are calculated for only six degrees of statistical error, namely, from 1 per cent. to 6 per cent. But it is easy to obtain the figures for any required degree, simply by dividing those in the column for 1 per cent. error by the square of the required degree. Thus for a probability of 99,999 to 1, and an observed percentage of 43, we must examine 478 things for a statistical error of 10 per cent., and 4,778,000 things for a statistical error of 1-10th per cent. It will be seen that the columns for 2 per cent., 3 per cent., etc., follow this rule.

It is also easy to obtain approximately the figures for fractional observed percentages, such as 2·5 per cent., or 27·3 per cent., because the increase in the number of things required to be examined is, roughly, proportional to the increase of the observed percentage. Subtract the next lower from the next higher figure in the table; multiply the remainder by the decimal fraction, and add the result to the next lower figure. Thus in Table A 1, in the column for 1 per cent., the figure opposite 2·5 per cent. observed percentage will be about 4,747; and opposite 27·3 per cent. will be about 38,686. But such refinements will rarely be needed.

We are often obliged to examine such a small number of things that the error is evidently greater than the 6 per cent. calculated for in the Tables, and we may wish to know exactly how much it is. In this case proceed as follows:—Select the degree of probability required, and in the appropriate table look out the observed percentage actually obtained. Take the number opposite to this in the column for 1 per cent. error, and divide it by the number of things actually examined. The error will be the square root of the quotient. Thus, suppose that we have examined only 300 leucocytes, and have found 45 per cent. of these to be mononuclears. The number we ought to examine for a 1 per cent. error at a betting of 9,999 to 1 is 37,459. The ratio of this number to 300 is 125; and the square root of this is 11·2. Hence the statistical error of our work at this betting is 11·2 per cent. But 6,697 leucocytes would have sufficed at a betting of only 9 to 1. The ratio of this to 300 is 22·32; so that the statistical error at 9 to 1 is only 4·7 per cent.—as could have been roughly inferred from Table A 4.

The reader should note the large number of things which must be examined before a result can be obtained to any high degree of probability and within narrow limits of error; and he will doubtless remember many confident assertions based upon much smaller samples.

EXAMPLES

1. How many persons of one nationality must be examined before we can assure ourselves, to a probability of 99,999 to 1, that from 65 per cent. to 67 per cent. of all the nation do not possess blue eyes? *Answer:* 43,744.

2. How many of a patient's leucocytes must be examined before we can bet 999 to 1 that between 42 per cent. and 40 per cent. of all his leucocytes are mononuclear?* *Answer:* 26,194.

3. How many of his leucocytes must be examined before we can bet *about* 100 to 1 that 69 per cent. to 65 per cent. of all his leucocytes are 'polynuclear'? *Answer:* 3,668.

4. How many of his leucocytes must be examined before we can bet 9,999 to 1 that his eosinophile leucocytes number between 1·5 per cent. and 2·5 per cent. of his total leucocytes? *Answer:* 11,868 (Multiply figure in column for 1 per cent. in Table A 2 by 4).

5. On examining 100 of his leucocytes we find that 7 per cent.

* Always supposing that the leucocytes are evenly distributed throughout the circulation.

of them are large mononuclears. How many *more* leucocytes must we examine before betting *about* 10,000 to 1 that the same percentage holds for all the leucocytes in his body within an error of 3 per cent.? *Answer:* 995.

6. On examining 65 case-records we find that 13 of the patients died. How many more case-records must we examine before betting 99 to 1 that the case-mortality of the disease is between 19 per cent. and 21 per cent? *Answer:* 10,551.

7. Our new line of treatment has cured one out of five cases of a hitherto incurable disease. How many more cases must we treat before betting 999 to 1 that we can cure between 15 per cent. and 25 per cent. of all cases? *Answer:* 688.

8. On examining nearly 2,000 of a patient's red corpuscles, we find that 8 per cent. of them are nucleated, and bet 9 to 1 that the percentage of nucleated red-corpuscles in his whole body is—what? *Answer:* Between 7 per cent. and 9 per cent.

9 On examining 220 things, we find that 31 of them belong to a particular class. What is the statistical error at a probability of 99 to 1? *Answer:* 6 per cent.

10. On examining 138 leucocytes we find that 15 per cent. of them are large mononuclears. What is the proportion of large mononuclears in the whole body, at a betting of about 1,000 to 1? *Answer:* Anything between 5 per cent. and 25 per cent. (Find the figure for 10 per cent. error.)

11. On examining 500 malaria parasites, we find that 14 per cent. of them are sexual forms. What is the betting that the statistical error is about 1 per cent.? *Answer:* An even chance.

12. In the same case, what is the betting that the error is not greater than 4 per cent.? *Answer:* 99 to 1.

13. Out of 200 leucocytes we find 23 per cent. to be mononuclears. What is the statistical error at a betting of 999 to 1? *Answer:* 9·79 per cent. (Square root of $\frac{19178}{200}$).

14. Next day, in the same patient, out of the same number of leucocytes, we find 40 per cent. to be mononuclears. What is the statistical error at the same betting? *Answer:* 11·4 per cent.

15. Can we bet 999 to 1 that there has been an increase of mononuclears in this case? *Answer:* No, because the errors overlap;

that is, the difference between 23 and 40 is less than the sum of 9·79 and 11·4.

16. Can we bet 99 to 1 that there has been an increase? *Answer:* Yes, because with this lower degree of probability the sum of the errors, namely 7·7 and 8·9, is less than the difference between the observed percentages, 23 and 40.

17. Working at a probability of 999 to 1, we find that out of 100 leucocytes two are eosinophiles. What is the error? *Answer:* Between 4 per cent. and 5 per cent. What are we to conclude? *Answer:* That we must examine more leucocytes until the error is at least less than the observed percentage.

18. On examining 136 things we find about 10 per cent. to belong to a particular class. What, roughly, is the error at a betting of 9,999 to 1? *Answer:* 10 per cent. (Find the square root of the quotient of 13,621 divided by 136.)

19. On examining 200 things we find 80 of them belong to a particular class. What is the error at a betting of 99,999 to 1? *Answer:* 15·3 per cent. (Divide 46,785 by 200, and find the square root of the quotient.)

20. Working at a probability of 9,999 to 1, and an error of 1 per cent., how many things must we examine in order to assure an observed percentage of 41·7? *Answer:* About 36,789.

I. THE CORRECT PROCEDURE IN PRACTICAL WORK

The Tables will, then, be of practical use in many kinds of sanitary and medical work; as, for instance, in estimating the frequency of death, or of some symptom in a given disease; or of some symptom, such as enlargement of the spleen or rickets, in a population; or in making differential counts of leucocytes in a patient, or of colonies of bacteria growing on a plate culture. But an examination of the Tables will convince us that the procedure now generally adopted in attempting such estimates is very faulty, because observers seldom trouble much regarding that all-important point, the *size* of the sample—that is, the total number of things which they must examine in order to obtain a sufficiently correct result. Certainly, often (though not always), they recognise that the sample must be large; but they usually fix its size quite arbitrarily—as, for instance, when they say beforehand that 200, or 500, or 1,000 leucocytes must be examined for differential counts. This, however, may lead to the most untrustworthy results, because, as we have seen, the size of the required sample is not fixed, but depends on several factors, including the observed percentage of things of the particular class—that is, the very percentage which we are seeking to ascertain. We cannot, therefore, fix the size of the sample beforehand, but must do so as we proceed in the work.

The size of the sample depends upon three factors, namely :

(1) The degree of sureness which we have to attain ;

(2) The percentage of statistical error, or degree of accuracy, which may be allowed ; and

(3) The observed percentage of things of the particular class which we are endeavouring to ascertain.

The *correct procedure* is, therefore, as follows :—

(1) First decide definitely as to the degree of sureness which must be attained, and the percentage of error which may be allowed. These will depend upon the importance of the work and the time which we can devote to it. For strict scientific or large sanitary investigations we may require a very high degree of sureness, say, 99,999 to 1, and very small limits of error, say 1 per cent. And this will be specially the case when we have to compare results obtained at different times ; as, for instance, when we wish to know whether the mononuclear leucocytes increase with the progress of a disease (see examples 13-16), or whether an epidemic is diminishing. Here it is absolutely essential that the statistical errors obtained at the two different times are not large enough to overlap. On the other hand, we may often be permitted to adopt lower degrees of sureness and high percentages of error, especially when we are merely seeking some corroborative evidence or when differences between successive estimates are so large and striking that even a large percentage of error cannot mislead the judgment. Here, as in regard to the following paragraph, we must often be guided by the progress of the work. But, as soon as we decide upon these points, we can determine which table, and which column in that table, are to be used.

(2) Secondly, before fixing upon the size of the sample, we should endeavour to obtain by trial a rough estimate of the observed percentage of things of the particular class which we are studying.

Suppose, for example, that we have to make a differential count of leucocytes. Then we do not wish, on the one hand, to allow too much statistical error, or, on the other hand, to waste time over examining too large a sample. Suppose, first, that the 9 to 1 Table is sufficient. Begin by examining 100 leucocytes. Suppose that 40 per cent. of these are mononuclears. Then we can see at once from the Table how many leucocytes must be examined to give a reliable result at that observed percentage. If a 6 per cent. error will suffice, we anticipate that we shall require to examine only 81 more leucocytes. If a 2 per cent. error must be obtained we shall have to examine 1,524 more leucocytes. As we now proceed in the task we shall find that the observed percentage changes considerably when we have examined 200, 300, 400 leucocytes, and so on (we should calculate the percentage, not for each successive batch of 100 leucocytes, but for the total number examined from the beginning). Finally, when about 1,400 leucocytes have been examined, we anticipate that we are approaching the required limit (for 2 per cent. error). Suppose that at 1,559 leucocytes the observed percentage stands at just about 36 per cent.—thus agreeing with the Table. We then stop; having obtained a 36 per cent. ratio, with a betting of 9 to 1, and a statistical error of 2 per cent. That is, the probability is 9 to 1 that the truth lies between 38 per cent and 34 per cent.; and we have not wasted time in reaching this result.

If, however, we require high degrees of probability, or low degrees of error, or both, there will be little use in attempting the preliminary rough estimate by a small sample of only 100 leucocytes, and we had better take for it at once 500 or 1,000 or more as the case might be.

(3) Of course, in differential leucocyte counts we often possess beforehand some inkling of what observed percentage we are to expect. Thus, the eosinophiles are generally few in number, and the 'polynuclears' numerous; and we judge roughly regarding the size of the sample accordingly. The same thing usually happens in other kinds of enquiry.

(4) It is a great mistake to suppose that a large sample will compensate for inaccurate working. These Tables are based on the supposition that each thing examined has been accurately assigned to its proper class. Things which cannot be certainly

assigned to their proper class must be rejected; but the number of them must be noted, and the proportion which they bear to the whole number of things must be afterwards determined, with estimates of probability and error, by precisely the same methods as those described. They constitute, in fact, a third class by themselves.

(5) If the things under study can be divided into three or more classes, determine separately the proportion of each class to the whole. This does not necessarily require different series of investigations. We simply extract the figures from the records; but care must be taken that the samples are sufficient for each class by itself.

(6) If while examining successive samples of things (such as leucocytes) we find that the observed percentage in each sample in succession tends always in one direction, that is, either to increase or to decrease, then we may *suspect* that some influence other than mere chance is at work. The number of successive samples required to verify such a suspicion will depend upon the nature of the material, and might be large; but in many cases if the observed percentage always increases, or always diminishes, in at least five successive samples, then we may have grounds for further enquiry upon the point, or for reference to a trained statistician.

(7) The probability or degree of betting which is generally accepted by statisticians as amounting almost to certainty is 49,999 to 1. The figures for this can be obtained by multiplying the corresponding figures for a betting of 9,999 to 1 (Table A 2) by the factor 1·189; or, what is nearly the same thing, by increasing the figures in Table A 2, by 20 per cent.

(8) Great care must always be taken that samples are chosen really at random, and are not selected with any conscious or subconscious bias.

3. PROPORTIONATELY LARGE SAMPLES

We have hitherto dealt with Comparatively Small Samples—that is, with samples which are small compared with the total number of the things in existence. For instance, if we are studying all the people in a country or all the leucocytes in a patient's body, we shall seldom be able to examine more than a very small proportion of these people or leucocytes. But there are cases when the total number of things is not so very large that we cannot examine a very considerable proportion of them. Suppose, for instance, that we wish to ascertain the proportion of children with enlarged spleen, not in the whole world, but in a large village; and suppose that there are 200 children in the village, but that we have time to examine only 50 of them. Here the sample is comparatively large, being one quarter of all the children in the village. Or suppose that we can examine 180 of the children; here the sample is so large that it approaches the whole number of things under study (i.e., the children in the village). Obviously, on examining these large samples we shall approach much nearer to the exact truth than would be anticipated from the Tables. In the second case, for instance, we should have to examine only 20 more of the children in order to obtain absolute certainty (provided that the examinations are careful enough). Hence, clearly, the Tables must be corrected for the case of Large Samples. A very suitable and easily applicable method for this purpose is to multiply the statistical error given in the Tables by the Factor.

$$\sqrt{1 - \frac{n-1}{N-1}}$$

where n is the number of things in the sample, and N is the total number of things under study.

Examining this Correction Factor, we see that when n is very small compared with N, the term $\frac{n-1}{N-1}$ becomes so small that it may be neglected; so that the Factor now becomes the square root of unity; that is, unity. This multiplied into the statistical errors given in the Tables does not modify them at all—so that the Tables are then quite correct. Here we have, of course, the case of Comparatively Small Samples, where N is a very large number, such as all the people in a country or all the leucocytes in a person.

Again, if $n = N$, that is, when the sample includes all the things in existence, the term $\frac{n-1}{N-1}$ equals unity, and the Factor becomes zero. This multiplied into the statistical errors makes them vanish. In other words, there is no statistical error because we have reached certainty.

Between these values the Factor is a vulgar fraction, which can easily be calculated. Thus, if $n = 50$ and $N = 200$, the Factor is the square root of $\frac{150}{199}$: that is, 0·868. This reduces the statistical error, but not much. If $n = 180$, the Factor is the square root of $\frac{20}{199}$: that is, 0·317—which reduces them considerably.

If the total number of things is at all considerable—say over 20—then the Factor becomes nearly the same as the square root of $1 - \frac{n}{N}$. In this case we can give a table of the various values of the Factor which correspond to the various values of $\frac{n}{N}$. In the following Table the proportion of things examined to total things, $\frac{n}{N}$, is given in percentages, and the corresponding values of the Factor are put below:—

Table B

$100\frac{n}{N}$	=	5	10	15	20	25	30	35	40	45	50
Factor	=	0·97	0·95	0·92	0·89	0·87	0·84	0·81	0·77	0·74	0·71
$100\frac{n}{N}$	=	55	60	65	70	75	80	85	90	95	100
Factor	=	0·67	0·63	0·59	0·55	0·50	0·45	0·39	0·32	0·22	0·00

Thus, if we examine half the total things, the statistical errors are reduced to about seven-tenths of the values given in Tables A. It will be seen that this Factor makes little difference in the statistical error unless the number of things examined is more than one-tenth of the total.

EXAMPLES

21. Nine out of 145 children in a village are absent. On examining the remainder we find 14 (say 10 per cent.) of them with enlarged spleen. What is the error at a betting of 9,999 to 1? *Answer:* 2·5 per cent. (The Factor is 1/4. See also Example 18.)

22. There are 1,000 people on an island. Out of 100 of these 60 are found to be affected with filariasis. What is the statistical error at a betting of 9 to 1? *Answer:* About 7·7 per cent.

23. Out of 884 people in a town, three-quarters are examined, and 73 of these are found to be in bad health. How many of all the people in the town are likely to be unwell? *Answer:* Between 9 per cent. and 13 per cent., at a betting of 999 to 1.

4. THE NUMBER OF THINGS OF ONE CLASS EXISTING IN A GIVEN AREA, BULK, OR TIME

Suppose that we wish to know the number of things of one kind contained in a given area, bulk, or time. Then the only way to ascertain this with certainty is to count all the things. Suppose, however, that we have no time for this, and must content ourself with counting the things in a measured sample, and then estimating from this observed result the most probable number of the things in the whole area, bulk or time. The question then arises: How large must the sample be in order to reduce the statistical error, with a given degree of probability, to below a given percentage?

For example, suppose that we wish to know how many separate stones there are in a million cubic feet of gravel. We cannot count them all, and must, therefore, content ourself with counting how many there are in a sample of, say, one, two or more cubic feet. In how many cubic feet, then, must we count the separate stones in order to be able to calculate the total with the required degree of accuracy? Or suppose that we wish to know how many leucocytes or parasites there are in the total blood of a patient, then in how much of his blood must we count these objects in order that the most probable truth will lie between sufficiently narrow limits? Shall we take one, two, or more cubic millimetres of his blood for our sample?

First, in order to use the sampling method at all, we must know that the things are equally distributed throughout the area, bulk, or time. If this is not the case, we cannot know that our sample accurately represents the whole material. For example, it would be useless to attempt to estimate the population of Britain by counting all the people in one-tenth of the area of the country because the population is not distributed equally at random everywhere, but is gathered specially into certain districts and cities according to certain economic laws. Similarly, we cannot estimate by taking samples of the peripheral blood how many blood parasites there are in the whole body unless we know that these organisms do not collect specially in certain parts of the circulation.

But—it may be asked—if the things are equably distributed, what further trouble will there be? We have only to count the number found in any sample, and then to multiply that figure by the total number of samples contained in the whole area, bulk, or time. If the stones are equally distributed in a million cubic feet of gravel, then there will be exactly a million times as many in the whole mass of gravel as there are in one cubic foot. But this is not so. It may be that *by chance* the stones in the first cubic foot taken as a sample are exceptionally large, and therefore are exceptionally few. Or it may happen by chance that the trypanosomes in a first sample of blood are exceptionally numerous, or exceptionally few, as the case may be. We shall then form a totally wrong estimate if we trust merely to the simple but untruthful method just mentioned.

To obtain accurate estimates by any method in cases like these may require the services of a trained statistician, and also, often, a special study of the kind of material under consideration—especially to ascertain whether the things are really equably distributed. But for the purposes of this Article the following Table will often be useful, because it serves to give some idea of the number of things which must actually be counted if they are equably distributed throughout the whole area, bulk, or time.

TABLE C

PROBABILITY	STATISTICAL ERRORS						
	1 %	2 %	3 %	4 %	5 %	10 %	20 %
1 to 1	4.550	1,138	506	285	182	46	11
9 to 1	27,057	6.764	3,006	1,691	1,082	271	68
99 to 1	66,350	16.588	7.372	4,147	2,654	664	166
999 to 1	108,284	27,071	12,032	6,768	4.331	1,083	271
9,999 to 1	151,338	37,835	16,815	9,459	6,053	1,513	378
99,999 to 1	194,938	48,735	21,660	12,184	7,797	1,949	487

Suppose, for example, that a cubic foot of gravel has been found to contain 3,006 stones, then the most probable number of stones in a million cubic feet of the same gravel will be 3,006 millions, within an error of 3 per cent., and at a betting of 9 to 1; that is, we may bet 9 to 1 that the most probable total number of stones in the million cubic feet will lie between about 3,096 and 2,916 millions. Or suppose that 271 trypanosomes have been found in one cubic millimetre of blood in a patient weighing 64·74 kilogrammes (142 lbs., or about 10 stone English), who should contain about 3,000,000 c.mm. of blood altogether; then we may bet 9 to 1 that the most probable number of trypanosomes in the whole of his blood will lie between 894,300,000 and 731,700,000 (10 per cent. error). And if we have counted 4,200 leucocytes in the c.mm. of blood, we may bet 9,999 to 1 that his total blood contains between 13,356 and 11,844 millions of these cells (6 per cent. error).

As stated in Section 1 (page 350) if we wish to know the number of things to be counted in order to yield an error within more than 5 per cent. we have only to divide the figures under the 1 per cent. column twice over by the required percentage (that is, by the square of the required percentage). Thus, for an error of 10 per cent. we divide by 100; and for an error of 31·6 per cent. we divide by 1,000. For example, if we find 27 malaria crescents in 1 c.mm. of the same patient's blood, we may bet 9 to 1 that he contains between about 106 and 56 millions of crescents altogether (always provided that they are equably distributed in the blood).

The above Table is only for *proportionately small samples*, as defined in Section 1 (page 348); that is, for samples which are small compared with the total mass of material. When the sample is more than about one-tenth of the total material we should use the Correction Factor of Section 3 (page 357) for *proportionately large samples*. Thus, if 1,500 leucocytes have been counted in one-quarter c.mm. of blood, and we wish to calculate the number in 1 c.mm., then the error by the above Table is about 10 per cent. at a betting of 9,999 to 1. But by the Table on page 358 the Correction Factor is 0·87 when $\frac{n}{N}$ equals one-quarter, or 25 per cent. Thus the error is not 10 per cent., but 8·7 per cent.; and we may bet 9,999 to 1 that the number of leucocytes in 1 c.mm. of blood is between 6,522 and 5,478. But for the total blood content of 3,000,000 c.mm. the error of 10 per cent. must be maintained, because the one-quarter c.mm. is now a 'proportionately small sample.' This gives the number of leucocytes in the whole body as most probably lying between 19,800 and 16,200 millions.

We can use the Table in another way. Suppose that we have counted 664 things in a sample. Then the error is 10 per cent. on a probability of 99 to 1. Hence we can bet 99 to 1 that all counts in future samples of the same size will lie between 730·4 and 597·6—assuming that the sample is proportionately small. This way of stating the case avoids the necessity of determining exactly the size of the sample compared to the whole material. In blood counts, for instance, we are concerned with the number of things in unit of blood rather than in the whole body, and we often wish to know whether this number is increasing or diminishing. If the number of things in a second sample is outside the limits of error declared from the first sample, we may assume, at the appropriate probability, that there has been an increase or decrease, as the case may be. If otherwise, the difference may be due merely to chance in the taking of the samples, and not to any real change in the total number of things in the whole body.

EXAMPLES

24. We have counted 4,250 red corpuscles in one-thousandth of a cubic millimetre of blood. What is the most probable number in one cubic millimetre, at a betting of 99 to 1? *Answer:* Between about 4,420,000 and 4,080,000.

25. How many may we expect to find in a second sample of the same size, at a betting of 999 to 1? *Answer:* Between about 4,463 and 4,037.

26. A week ago we found 490 red corpuscles in one-ten-thousandth of a c.mm of a patient's blood. To-day we find only 294 in the same sized sample. May we bet 99,999 to 1 that there has been a decrease? *Answer:* No, the errors overlap. May we bet 9 to 1 that there has been a decrease? *Answer:* Yes.

27. Blood has been diluted 100 times. In $\frac{1}{100}$th of 1 c.mm. of the mixture we found 500 red corpuscles. How many do we expect to find in 1 c.mm. of the blood at a probability of 999 to 1? *Answer:* 5,000,000, with an error of 14·7 per cent.

28. We have found 553 things in a sample. In how many out of 100 similar samples of the same material should we expect to find an error greater than 7 per cent.? *Answer:* In ten.

29. A newly-appointed official finds that he is obliged to write

150 letters during his first week of office. How many letters should he expect, at a betting of 999 to 1, to have to write at the same rate every week in the future? *Answer:* 150, with an error of 26·87 per cent.

30. We have found one filaria embryo in 1 c.mm. of blood. What may we infer regarding the total number in the whole circulation of 3,000,000 c.mm.? *Answer:* The odds are 1 to 1 that the error is less than 67·5 per cent., and that the total number of embryos in the circulation lies between 5,025,000 and 975,000. In one out of two such cases the error may exceed this amount.

II. TABLES

A 1. 99999 to 1

%	Errors 1 %	2 %	3 %	4 %	5 %	6 %
1	1930	483	215	121	78	54
2	3821	956	425	239	153	107
3	5673	1419	631	355	227	158
4	7486	1872	832	468	300	208
5	9260	2315	1029	579	371	258
6	10995	2749	1222	688	440	306
7	12691	3173	1410	794	508	353
8	14348	3587	1595	897	574	399
9	15966	3992	1774	998	639	444
10	17545	4387	1950	1097	702	488
11	19085	4772	2121	1193	764	531
12	20586	5147	2288	1287	824	572
13	22048	5512	2450	1378	882	613
14	23471	5868	2608	1467	939	652
15	24855	6214	2762	1554	995	691
16	26200	6550	2912	1638	1048	728
17	27506	6877	3057	1720	1101	765
18	28773	7194	3197	1799	1151	800
19	30000	7500	3334	1875	1200	834
20	31191	7798	3466	1950	1244	867
21	32340	8085	3594	2022	1294	899
22	33451	8363	3717	2091	1339	930
23	34524	8631	3836	2158	1381	959
24	35557	8889	3951	2223	1423	988
25	36551	9138	4062	2285	1463	1016
26	37506	9377	4168	2345	1501	1042
27	38423	9606	4270	2402	1537	1068
28	39300	9825	4367	2457	1572	1092
29	40138	10035	4460	2509	1606	1115
30	40936	10234	4549	2559	1638	1138
31	41698	10425	4634	2607	1668	1159
32	42419	10605	4714	2652	1697	1179
33	43101	10776	4789	2694	1724	1198
34	43744	10936	4861	2734	1750	1216
35	44349	11088	4928	2772	1774	1232
36	44914	11229	4991	2808	1797	1248
37	45440	11360	5049	2840	1818	1263
38	45928	11482	5104	2871	1837	1276
39	46366	11592	5152	2898	1855	1288
40	46785	11697	5199	2925	1872	1300
41	47156	11789	5240	2948	1887	1310
42	47487	11872	5277	2968	1900	1320
43	47780	11945	5309	2987	1912	1328
44	48033	12009	5337	3003	1922	1335
45	48247	12062	5361	3016	1930	1341
46	48423	12106	5381	3027	1937	1346
47	48559	12140	5396	3035	1943	1349
48	48657	12165	5407	3042	1947	1352
49	48715	12179	5413	3045	1949	1354
50	48735	12184	5415	3046	1950	1354

A 2. **9999 to 1**

%	Errors 1 %	2 %	3 %	4 %	5 %	6 %
1	1499	375	167	94	60	42
2	2967	742	330	186	119	83
3	4404	1101	490	276	177	123
4	5812	1453	646	364	233	162
5	7189	1798	799	450	288	200
6	8536	2134	949	534	342	238
7	9852	2463	1095	616	395	274
8	11139	2785	1238	697	446	310
9	12395	3099	1378	775	496	345
10	13621	3406	1514	852	545	379
11	14816	3704	1647	926	593	412
12	15982	3996	1776	999	640	444
13	17117	4280	1902	1070	685	476
14	18221	4556	2025	1139	729	507
15	19296	4824	2144	1206	772	536
16	20340	5085	2260	1272	814	565
17	21354	5339	2373	1335	855	594
18	22338	5585	2482	1397	894	621
19	23291	5823	2588	1456	932	647
20	24215	6054	2691	1514	969	673
21	25107	6277	2790	1570	1005	698
22	25970	6493	2886	1624	1039	722
23	26802	6701	2978	1676	1073	745
24	27605	6902	3068	1726	1105	767
25	28376	7094	3153	1774	1135	789
26	29118	7280	3236	1820	1165	809
27	29829	7458	3315	1865	1193	829
28	30510	7628	3390	1907	1221	848
29	31160	7790	3463	1948	1247	866
30	31781	7946	3532	1987	1272	883
31	32372	8093	3597	2024	1295	900
32	32932	8233	3660	2059	1317	915
33	33461	8366	3718	2092	1339	930
34	33960	8490	3774	2123	1359	944
35	34430	8608	3826	2152	1378	957
36	34868	8717	3875	2180	1395	969
37	35277	8820	3920	2205	1411	980
38	35655	8914	3962	2229	1427	991
39	36004	9001	4001	2250	1441	1000
40	36322	9081	4036	2270	1453	1009
41	36609	9153	4068	2289	1465	1017
42	36866	9217	4097	2305	1475	1025
43	37093	9274	4122	2319	1484	1031
44	37290	9323	4144	2331	1492	1036
45	37459	9365	4162	2342	1499	1041
46	37593	9399	4177	2350	1504	1045
47	37699	9425	4189	2357	1508	1048
48	37774	9444	4198	2361	1511	1050
49	37820	9455	4203	2364	1513	1051
50	37835	9459	4204	2365	1514	1051

A 3. 999 to 1

%	Errors 1 %	2 %	3 %	4 %	5 %	6 %
1	1072	268	120	67	43	30
2	2123	531	236	133	85	59
3	3152	788	351	197	127	88
4	4159	1040	463	260	167	116
5	5144	1286	572	322	206	143
6	6107	1527	679	382	245	170
7	7050	1763	784	441	282	196
8	7970	1993	886	499	319	222
9	8869	2218	986	555	355	247
10	9746	2437	1083	610	390	271
11	10601	2651	1178	663	424	295
12	11435	2859	1271	715	458	318
13	12247	3062	1361	766	490	341
14	13038	3260	1449	815	522	363
15	13807	3452	1535	863	553	384
16	14554	3639	1617	910	583	405
17	15279	3820	1698	955	611	425
18	15983	3996	1776	999	640	444
19	16665	4167	1852	1042	667	463
20	17326	4332	1925	1083	693	482
21	17965	4492	1995	1123	719	499
22	18582	4646	2065	1162	744	517
23	19178	4795	2131	1199	767	533
24	19751	4938	2195	1235	791	549
25	20304	5076	2256	1269	813	564
26	20834	5209	2315	1303	833	579
27	21343	5336	2372	1334	854	593
28	21830	5458	2426	1365	874	607
29	22296	5574	2478	1394	892	620
30	22740	5685	2527	1422	910	632
31	23162	5791	2574	1448	927	644
32	23562	5891	2618	1473	943	655
33	23942	5986	2661	1497	958	666
34	24299	6075	2700	1519	972	675
35	24635	6159	2738	1540	986	685
36	24949	6238	2773	1560	998	694
37	25241	6311	2805	1578	1010	702
38	25512	6378	2835	1595	1021	709
39	25761	6441	2863	1611	1031	716
40	25989	6498	2888	1625	1040	722
41	26194	6549	2911	1638	1048	728
42	26378	6595	2931	1649	1055	733
43	26541	6636	2949	1659	1062	738
44	26682	6671	2965	1668	1068	742
45	26800	6700	2978	1675	1072	745
46	26898	6725	2989	1682	1076	748
47	26974	6744	2997	1686	1079	750
48	27028	6757	3004	1689	1081	751
49	27061	6766	3007	1692	1082	751
50	27071	6768	3008	1692	1083	752

A4. 99 to 1

%	Errors 1 %	2 %	3 %	4 %	5 %	6 %
1	658	165	74	42	27	19
2	1301	325	145	82	52	37
3	1931	483	215	121	78	54
4	2549	638	284	160	102	71
5	3151	788	350	197	126	88
6	3743	936	416	234	150	104
7	4319	1080	480	270	173	120
8	4884	1221	543	306	196	136
9	5434	1359	604	340	218	151
10	5973	1494	664	374	239	166
11	6496	1624	728	406	260	181
12	7007	1752	779	438	281	195
13	7504	1876	834	469	301	209
14	7989	1998	888	500	320	222
15	8461	2116	940	529	339	236
16	8918	2230	991	558	357	248
17	9361	2341	1040	586	375	260
18	9794	2449	1089	613	392	273
19	10211	2553	1135	639	409	284
20	10616	2654	1180	664	425	295
21	11007	2752	1223	688	441	306
22	11386	2847	1265	712	456	317
23	11752	2938	1306	735	470	327
24	12104	3026	1345	757	485	337
25	12441	3111	1383	778	498	346
26	12767	3192	1419	798	511	355
27	13078	3270	1453	818	523	364
28	13376	3344	1487	836	535	372
29	13682	3421	1521	856	548	381
30	13934	3484	1549	871	558	388
31	14193	3549	1577	888	568	395
32	14439	3610	1605	903	578	402
33	14672	3668	1631	917	587	408
34	14890	3723	1655	931	596	414
35	15096	3774	1678	944	604	420
36	15287	3822	1699	956	612	425
37	15467	3867	1719	967	619	430
38	15633	3909	1737	978	625	435
39	15787	3947	1755	987	632	439
40	15925	3982	1770	996	637	443
41	16051	4013	1784	1004	642	446
42	16164	4041	1796	1011	647	449
43	16263	4066	1807	1017	651	452
44	16349	4088	1817	1022	654	455
45	16422	4106	1825	1027	657	457
46	16482	4121	1832	1031	660	458
47	16530	4133	1837	1034	662	460
48	16562	4141	1841	1036	663	461
49	16582	4146	1843	1037	664	461
50	16588	4147	1844	1037	664	461

A5. 9 to 1

%	Errors 1 %	2 %	3 %	4 %	5 %	6 %
1	268	67	30	17	11	8
2	531	133	59	34	22	15
3	788	197	88	50	32	22
4	1039	260	116	65	42	29
5	1286	322	143	81	52	36
6	1526	382	170	96	62	43
7	1762	441	196	111	71	49
8	1992	498	222	125	80	56
9	2216	554	247	139	89	62
10	2436	609	271	153	98	68
11	2649	663	295	166	106	74
12	2858	715	318	179	115	80
13	3061	766	341	192	123	86
14	3258	815	362	204	131	91
15	3450	863	384	216	139	96
16	3637	910	405	228	146	102
17	3818	955	425	239	153	107
18	3994	999	444	250	160	111
19	4164	1041	463	261	167	116
20	4330	1083	482	271	174	121
21	4489	1123	499	281	180	125
22	4643	1161	516	291	186	129
23	4792	1198	533	300	192	134
24	4936	1234	549	309	198	138
25	5074	1269	564	318	203	141
26	5206	1302	579	326	209	145
27	5333	1334	593	334	214	149
28	5455	1364	607	341	219	152
29	5571	1393	619	349	223	155
30	5682	1421	632	356	228	158
31	5788	1447	644	362	232	161
32	5888	1472	655	368	236	164
33	5983	1496	665	374	240	167
34	6072	1518	675	380	243	169
35	6156	1539	684	385	246	171
36	6234	1559	693	390	250	174
37	6307	1577	701	395	253	176
38	6375	1594	709	399	255	178
39	6437	1610	716	403	258	179
40	6494	1624	722	406	260	181
41	6545	1637	728	410	262	182
42	6591	1648	733	412	264	184
43	6632	1658	737	415	266	185
44	6667	1667	741	417	267	186
45	6697	1675	745	419	268	187
46	6721	1681	747	421	269	187
47	6740	1685	749	422	270	188
48	6754	1689	751	423	271	188
49	6762	1691	752	423	271	188
50	6764	1691	752	423	271	188

A 6. 1 to 1

%	Errors 1 %	2 %	3 %	4 %	5 %	6 %
1	45	11	5	3	2	1
2	89	22	10	6	4	2
3	132	33	15	8	5	4
4	175	44	19	11	7	5
5	216	54	24	14	9	6
6	257	64	29	16	10	7
7	296	74	33	19	12	8
8	335	84	37	21	13	9
9	373	93	41	23	15	10
10	409	102	45	26	16	11
11	445	111	49	28	18	12
12	480	120	53	30	19	13
13	515	129	57	32	21	14
14	548	137	61	34	22	15
15	580	145	64	36	23	16
16	611	153	68	38	25	17
17	642	160	71	40	26	18
18	671	168	75	42	27	19
19	700	175	78	44	28	19
20	728	182	81	45	29	20
21	755	186	84	47	30	21
22	781	195	87	48	31	22
23	806	201	90	50	32	22
24	830	207	92	52	33	23
25	853	213	95	53	34	24
26	875	219	97	55	35	24
27	897	224	100	56	36	25
28	917	229	102	57	37	25
29	937	234	104	59	37	26
30	955	239	106	60	38	27
31	973	243	108	61	39	27
32	990	247	110	62	40	28
33	1006	251	112	63	40	28
34	1021	255	113	64	41	28
35	1035	259	115	65	41	29
36	1048	262	116	66	41	29
37	1060	265	118	66	42	29
38	1072	268	119	67	42	30
39	1082	271	120	68	43	30
40	1092	273	121	68	43	30
41	1100	275	122	69	44	31
42	1108	277	123	69	44	31
43	1115	279	124	70	44	31
44	1121	280	125	70	45	31
45	1126	281	125	70	45	31
46	1130	282	126	71	45	31
47	1133	283	126	71	45	32
48	1135	284	126	71	45	32
49	1137	284	126	71	45	32
50	1137	284	126	71	45	32